Prescribed Curvature Problems in Two Dimensions

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Abstract

The Uniformisation Theorem tells us that there exists for every metric $g_0$ on a closed two-dimensional Riemannian manifold $M$ a metric $\bar{g}$ which is pointwise conformal to $g_0$ and has a constant Gauss curvature $\bar{K} \equiv \bar{K} \in \mathbb{R}$.

More generally, we may ask if there exists for a given function $f \in C^\infty(M)$ on this manifold a conformal metric $g$ with Gauss curvature $K_g = f$. This is the so-called prescribed Gauss curvature problem, which we want to study in particular on manifolds with negative Euler characteristic in the case of sign-changing functions $f$.

One way to construct solutions of this prescribed Gauss curvature problem is to see it as an energy minimisation problem and consider the corresponding gradient flow. This leads us to the prescribed Gauss curvature flow, which becomes the Ricci flow when looking for a constant curvature metric.

We will study solutions of this flow and analyse under which conditions they converge to a solution of the static problem. As in the case of the related harmonic map flow where reverse bubbling phenomena lead to a non-uniqueness of weak solutions, we investigate which weak solution concepts for the Ricci flow admit also such kind of bubbling behaviour.