Existence of immersed spheres minimizing curvature functionals in compact 3-manifolds

Abstract: We study curvature functionals for immersed 2-spheres in a compact, three-dimensional Riemannian manifold M. Under the assumption that the sectional curvature K^M is strictly positive, we prove the existence of a smooth immersion $f : \mathbb{S}^2 \to M$ minimizing the L^2 integral of the second fundamental form. Assuming instead that $K^M \leq 2$ and that there is some point $\overline{x} \in M$ with scalar curvature $R^M(\overline{x}) > 6$, we obtain a smooth minimizer $f : \mathbb{S}^2 \to M$ for the functional $\int \frac{1}{4}|H|^2 + 1$, where H is the mean curvature.

This is joint work with Ernst Kuwert and Andrea Mondino.