

Given a closed surface Σ immersed in a Riemannian manifold (M, g) of dimension 3, the Willmore functional W_1 is defined as

$$W_1(\Sigma) = 1/4 \int_{\Sigma} H^2 dS$$

where H is the mean curvature ($H = k_1 + k_2$ where k_1, k_2 are the principal curvatures) and dS the area form given by the immersion. It is also interesting to study a slight modification of W_1 : let us define

$$W_2(\Sigma) = \int_{\Sigma} (H^2/4 - D) dS$$

where $D = k_1 k_2$. W_2 has the important property of being conformal invariant and will be called conformal Willmore functional.

The critical points of W_1 (resp. W_2) are called (resp. conformal) Willmore surfaces and the aim of the seminar is to study the existence of such surfaces. The topic is classical and has many applications (general relativity, biology, elasticity theory...); after an introduction about the employed method (it is performed a finite dimensional reduction) we will study the functional in a perturbative setting.