AG algebraic geometry   **WS08/09**
de Jong's resolution of singularities and applications

Main reference is [dJ]. [B1] also contains all proofs, [dJ2] is an improved result. The texts of Oort can be used as introduction. Essential ingredient is the projectivity of the moduli stack of stable curves. The original reference is [DM] and [K]. In [HM] the result is shown with geometric invariant theory.

0. Definition of alterations [dJ] 2.20, statement of main result [dJ] Thm 4.1 and Thm 6.5, [dJ2] Sketch of the method of proof e.g. as in introduction of [dJ]. Discussion of the program and distribution of talks.

1. Geometry of semi-stable curves: definition, dualizing sheaf, genus, automorphisms, [dJ] 2.21-2.23, see also [DM] 1.1-1.2 + cor. p. 78, [K] II, p. 162–163 etc.

2. Modifications of semi-stable curves, Lemma [dJ] 3.2, i.e. all of §3.

3. Formulation of the main result and first reductions, [dJ] 4.1-4.12,

4. Definition of Stacks as in [DM], statement of results on moduli stacks of curves covering [dJ] 2.24. See also [HM] 2 C, pp. 46–52 including a definition of the Hilbert scheme and its relation to $\mathcal{M}_{g,n}$. The starting sections of [DM], [K] are also good references.

5. Sketch of proof of projectivity of the moduli stack of stable curves with GIT, [HM] Chapter 4.

6. Application of the results on semi-stable curves,[dJ] 4.13-4.17

7. Rest of the proof of the main theorem [dJ] 4.18-4.28

The following talks should be seen as possible alternatives. We can do what and as much we feel like.

A. (2 talks) The relative and the arithmetic case, [dJ] §§5,6,8

B. The equivariant case, [dJ]§7

C. (1-2 talks, uses A) non-negativity of intersection numbers following Gabber,[B1] §6.1, [R] gives more details, further results in [KR].

D. weak resolution of singularities in characteristic 0 [AdJ]

E. Finiteness of rigid cohomology [B1] 6.2,[B2]

F. Semistability of monodromy operation in étale cohomology [B1] 6.3

G. Comparison isomorphism between crystalline and étale cohomology

H. Application in anabelian geometry [P]

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