Theory of Non-commutative Algebras

Freiburg WS 2024

The seminar in winter semester 2024/25 will take place on Friday from 8 to 10 in SR 404 Ernst-Zermelo-Str. 1. You will find a few suggestions at the end of this program.

The presentations should last 80 minutes so that there is time for a feedback session. An appropriate length for a talk without an audience is 70 minutes.

1 Introduction

In this seminar, we are going to study finite dimensional (unital, possibly non-commutative) algebras over a (commutative) ring $\mathbf{k}$. Prototypes are the rings square matrices over $\mathbf{k}$, finite field extensions, or the algebra $\mathbf{k}^n$ with diagonal multiplication.

We will concentrate on path algebras of finite quivers (German: Köcher). Modules over them are equivalently described as representations of the quiver. Many algebraic properties can be directly understood from properties of the quiver.

2 Talks

Talk 1 (18.10.2024)**.**

Introduce the basic notion about rings, algebras (over $\mathbf{k}$) and modules which will be used throughout the whole seminar. Definitions of rings, algebras, ideals, (left/right/bi) modules and homomorphisms. The talk should cover [Sch14, §4.1 Definition 4.1-3 Lemma 4.1, Corollary 4.2, §4.2 Definition 4.4, §4.3 Definition 4.8-10, Proposition 4.8].

Talk 2 (25.10.2024)**.**

Introduce more notions and results on rings and modules about endomorphism rings, local rings and the decomposability of modules. The talk should cover [Sch14, §4.5 Lemma 4.15, Corollary 4.20].

Talk 3 (08.11.2024)**.**

Introduce the definition of the quiver, basic notions of path algebras, modules of path algebras and the idempotent decomposition. Describe basic properties of path algebras. The talk should include [Sch14, §1.1, Definition 1.1, §4.2 Definition 4.5, §4.4]

Talk 4 (15.11.2024)**.**

Introduce the basic notion of the category of modules, equivalence of categories and criteria on Morita equivalence.

Talk 5 (22.11.2024)**.**

Introduce the basic notion of quiver representations, show the equivalence between the representations of the quiver and the modules of the corresponding path algebra. Describe some relationships between the shape of the quiver and the representations of the quiver. [Sch14, §1.1, §5.2 Theorem 5.4]

Talk 6 (29.11.2024)**.**

Introduce some notion of homological algebra, including exactness and $\mathrm{Hom}, \mathrm{Ext}$ functors. [Sch14, §1.4 §2.4, §4.3 Lemma 4.11]

Talk 7 (06.12.2024)**.**
Describe the projective representations and irreducible representations of the quiver. Present the standard projective resolution of representations. [Sch14, §2.1, §2.2]

Talk 8 (13.12.2024)**.**
Proof of Wedderburn–Artin theorem.

Talk 9 (20.12.2024)**.**
Quiver varieties and orbits [Sch14, §8.1].

Talk 10 (10.01.2025)**.**
Quadratic forms of the quiver [Sch14, §8.2]

Talk 11 (17.01.2025)**.**
Roots of Dynkin quivers [Sch14, §8.3].

Talk 12 (24.01.2025)**.**
Proof of the Gabriel's theorem (classification theorem), part I.

Talk 13 (31.01.2025)**.**
Proof of the Gabriel's theorem (classification theorem), part II.

Literatur

[AF98] F.W. Anderson and K.R. Fuller. *Rings and Categories of Modules*, volume 13 of *Graduate Texts in Mathematics*. Springer New York, NY, 1998.

[Sch14] R. Schiffler. *Quiver Representations*. CMS Books in Mathematics. Springer Cham, 2014.