

4 Problems II

Problem 4.1. Show the de Rham Lemma $H(A \otimes_{\mathbb{R}} C) \xrightarrow{\sim} \mathbf{Sm}(HC)$. Hint: Construct equivalences $\underline{HC} \xrightarrow{\sim} H(A \otimes_{\mathbb{R}} C)$ and $\underline{HC} \xrightarrow{\sim} \mathbf{Sm}(HC)$

Problem 4.2. Show that $Z^n(\mathcal{A}) \rightarrow \sigma^{\geq n} \mathcal{A}$ represents the sheafification of $Z^n(\mathcal{A})$ in $\mathbf{Fun}(\mathbf{Mf}, \mathbf{Ch}[W^{-1}])$.

Problem 4.3. Verify the basic exact sequences of differential cohomology.

Problem 4.4. Show that algebraic K -theory splits exact sequences of bundles (give details for the proof of Lemma 2.8).

Problem 4.5. Calculate the class predicted in Corollary 2.10 in the case of complexes on S^1

$$\mathcal{V} : 0 \rightarrow S^1 \times R \xrightarrow{\text{id}} S^1 \times R \rightarrow 0 \rightarrow 0 ,$$

$$\mathcal{V}' : 0 \rightarrow S^1 \times R \xrightarrow{\lambda} S^1 \times R \rightarrow 0 \rightarrow 0 ,$$

where $\lambda \in R^*$ is a unit.

Problem 4.6. Show that the Becker-Gottlieb transfer satisfies $\mathbf{tr} = 0$ if $T^v \pi$ admits a nowhere vanishing section.