# 3 Problems

**Problem 3.1.** *Let $L \to \mathbb{CP}^n$ be the tautological line bundle and $\nabla$ the connection on $L$ on the tautological line bundle as in the lecture. Let $\omega \in \mathcal{A}^2(\mathbb{CP}^n)$ be the unique harmonic form satisfying $\int_{\mathbb{CP}^1} \omega = 1$. Show that the curvature of $\nabla$ satisfies*

$$R^\nabla = 2\pi i \omega.$$

**Problem 3.2.** *We consider $\lambda \in \mathbb{C}^*$ and define the action of $\mathbb{Z}$ on the trivial bundle $\mathbb{R} \times \mathbb{C} \to \mathbb{R}$ by $n(t, z) \mapsto (t + n, \lambda^n z)$. This action preserves the trivial connection $\nabla^{triv}$. We let $V(\lambda) \to S^1$ be the quotient with the induced flat connection $\nabla(\lambda)$. Show that*

$$\omega(\nabla(\lambda)) = -2 \log |\lambda| \mathtt{or}_{S^1} \in H^{odd}_{dR}(S^1) \ .$$

**Problem 3.3.** *Show that one can identify $KU^0(\mathbb{CP}^n) \cong \mathbb{Z}[z]/(z^{n+1})$ as rings, where $z := \mathtt{cycl}(L) - 1$.*

**Problem 3.4.** *Let $M$ be a manifold, $I$ the interval $[0, 1]$ and $i_0, i_1 \colon M \hookrightarrow I \times M$ be the inclusions at the endpoints. Prove the following homotopy formula: If $\hat{x} \in \widehat{KU}^0(I \times M)$, then*

$$i_0^*(\hat{x}) - i_1^*(\hat{x}) = a(\int_{I \times M/M} R(\hat{x}))$$

*where $\int_{I\times M/M} \colon \mathcal{A} \otimes \mathbb{R}[b, b^{-1}](I \times M) \to \mathcal{A} \otimes \mathbb{R}[b, b^{-1}](M)[-1]$ is given by integration along the interval $I$.*

**Problem 3.5.** *Let $(V(\lambda), \nabla(\lambda))$ be as in Problem 3.2. Calculate $\widehat{KU}^0(S^1)$ and characterize the element $\widehat{\mathtt{cycl}}(V(\lambda), \nabla(\lambda))$.*