5 Problems III

Problem 5.1 *Show that there is a quasi-isomorphism*

$$\mathbf{IDR}(p) \to \mathtt{Cone}\left((2\pi i)^p \mathcal{A}_{\mathbb{R}} \oplus \mathcal{F}^p\mathcal{A} \to \mathcal{A}\right)[2p-1]$$

given by $\omega \mapsto \left(\omega|_0 \oplus \omega|_1, -\int_{I\times(\_)/(\_)} \omega\right)$. *Deduce that, if* $X_{\mathbb{Q}}$ *is projective, we have*

$$H^n(\mathbf{IDR}(p)(* \times X)) \cong H^{n+2p}_{AH}(X_{\mathbb{R}}, \mathbb{R}(p)) \quad \text{for } n \leq 0.$$

Problem 5.2 *Let* $\mathcal{C}$ *be an* ∞*-category which has all small colimits. Show that there exists a functor*

$$\bar{\mathbf{s}}\colon \mathbf{Fun}(\mathbf{Mf}^{op}, \mathcal{C}) \to \mathbf{Fun}(\mathbf{Mf}^{op}, \mathcal{C})$$

which on objects $M \in \mathbf{Mf}$ *is given by*

$$\bar{\mathbf{s}}(F)(M) = \mathtt{colim}_{\Delta^{op}} F(\Delta^\bullet \times M).$$

Problem 5.3 *Let* A *be a commutative ring. Interpreting every set as a discrete category, we view* A *as a semiring object in* $\mathbf{Cat}[W^{-1}]$. *On the other hand, we may also view* A *as a commutative dga concentrated in degree 0. Show that in* $\mathbf{CommMon}(\mathbf{Sp})$ *there is a natural equivalence*

$$K(A) \cong H(A).$$