Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 11.1.2013

Dozenten: PD Dr. Markus Junker, Prof. Dr. Martin Ziegler

Assistent: Dr. Juan Diego Caycedo

Tutor: Christoph Bier B.Sc.

Blatt 10

Aufgabe 1.

1. Suppose $\mathfrak{M}$ is countable and ω-saturated. Show that for all $a, b \in M^n$, if $\mathrm{tp}(a) = \mathrm{tp}(b)$ then there is an automorphism σ of $\mathfrak{M}$ such that $\sigma(a) = b$.
2. Let $\mathfrak{M}$ be countable and ω-saturated. Suppose $X \subseteq M^n$ is invariant under all automorphisms of $\mathfrak{M}$, i.e. for every automorphism σ of $\mathfrak{M}$ the set $\sigma(X) := \{\sigma(x) : x \in X\}$ is equal to X. Show that there is a collection of types in $S_n(T)$ such that X is the union of their sets of realizations in $\mathfrak{M}$.
3. Suppose $\mathfrak{M}$ is countable and $\aleph_0$-categorical. Show that if $X \subseteq M^n$ is invariant under all automorphisms of $\mathfrak{M}$, then X is definable.

Aufgabe 2. Let $\mathfrak{M}$ be a structure and assume that for some n only finitely many n-types are realised in $\mathfrak{M}$. Then any structure elementarily equivalent to $\mathfrak{M}$ satisfies exactly the same n-types.

Aufgabe 3. Prove the following:

1. If T is $\aleph_0$-categorical, then in any model $\mathfrak{M}$ the algebraic closure of a finite set is finite. (In particular, $\mathfrak{M}$ is *locally finite*, i.e., any substructure generated by a finite subset is finite.)
2. There is no $\aleph_0$-categorical theory of fields, i.e., if T is a complete $\mathrm{L}_{\mathrm{Ring}}$-theory containing Field, then T is not $\aleph_0$-categorical.

Aufgabe 4. Show that for every $n > 2$ there is a countable complete theory with exactly n countable models.

Hint: Consider $(\mathbb{Q}, <, P_0, \ldots, P_{n-2}, c_0, c_1, \ldots)$, where the P_i form a partition of $\mathbb{Q}$ into dense subsets and the c_i are an increasing sequence of elements of P_0.)

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[0] `http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/`