Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 25.1.2013
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Blatt 12

Aufgabe 1. By induction on n, using a non-principal ultrafilter on A, prove *Ramsey's theorem*:

Let A be an infinite set and n, k be positive integers. Let γ be a function from the set $[A]^n$ of n-element subsets of A into $\{1, \ldots, k\}$. Then there is an infinite subset B of A such that the restriction of γ to $[B]^n$ is a constant function.

Hint: Fix a non-principal ultrafilter $\mathcal{U}$ on A. For each $s \in [A]^{n-1}$ choose $c(s)$ such that $\{a \in A \mid \gamma(s \cup \{a\}) = c(s)\}$ belongs to $\mathcal{U}$. Construct a sequence $a_0, a_1, \ldots$ of distinct elements such that $\gamma(s \cup \{a_i\}) = c(s)$ for all $s \in [\{a_0, \ldots, a_{i-1}\}]^{n-1}$. Apply the induction hypothesis to c restricted to $[\{a_0, a_1, \ldots\}]^{n-1}$.

Aufgabe 2. Show that a theory with a definable linear order of the universe (like DLO and RCF) cannot be κ-stable for any κ.

You may use the following fact: There is a linear ordering of bigger cardinality than κ which has a dense subset of cardinality κ (This is Exercise 8.2.8 in the Tent-Ziegler book).

Aufgabe 3. Show that the theory of an equivalence relation with two infinite classes has quantifier elimination and is ω-stable. Is it $\aleph_1$-categorical?

Aufgabe 4. If $\mathfrak{A}$ is κ-saturated, then all definable subsets are either finite or have cardinality at least κ.

[0] `http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/`