

### Blatt 13

**Aufgabe 1.** If  $T$  is an  $L$ -theory and  $K$  is a sublanguage of  $L$ , the *reduct*  $T \upharpoonright K$  is the set of all  $K$ -sentences which follow from  $T$ . Show that a complete theory  $T$  is totally transcendental if and only if  $T \upharpoonright K$  is  $\omega$ -stable for all at most countable  $K \subseteq L$ .

**Aufgabe 2.** Prove that for arbitrary  $T$  if the isolated types are dense in all  $S_1(A)$ , then the isolated types are dense in all  $S_n(A)$ .

**Aufgabe 3.** Prove that for every countable  $T$  the following are equivalent:

- (a) Every parameter set has a prime extension. (We say that  $T$  has *prime extensions*.)
- (b) Over every countable parameter set the isolated types are dense.
- (c) Over every parameter set the isolated types are dense.

**Aufgabe 4.** 1. Let  $\pi: X \rightarrow Y$  be a continuous open map between topological spaces. Show that a point  $x \in X$  is isolated if and only if  $\pi(x)$  is isolated in  $Y$  and  $x$  is isolated in  $\pi^{-1}(\pi(x))$ .

2. Use part 1. to give a proof of the following lemma from the lectures: Let  $a$  and  $b$  be two finite tuples of elements of a structure  $\mathfrak{M}$ . Then  $\text{tp}(ab)$  is atomic if and only if  $\text{tp}(a/b)$  and  $\text{tp}(b)$  are atomic.