Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 8.2.2013
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Blatt 14

Aufgabe 1.

1. Adapt the proof of Lachlan's theorem to yield the following: Let T be totally transcendental and $\mathfrak{M}$ an uncountable model of T. Let $\mu \leq |M|$ be an uncountable regular cardinal. Then $\mathfrak{M}$ has arbitrarily large elementary extensions which omit every set of $L(M)$-formulas of size less than μ that is omitted in $\mathfrak{M}$.

2. Use part 1. to show that if a countable theory T is κ-categorical for some uncountable κ, then it is λ-categorical for every uncountable $\lambda \leq \kappa$.

Aufgabe 2. Prove that if T is totally transcendental and has a Vaughtian pair for $\varphi(x)$, then it has, for all uncountable κ, a model $\mathfrak{M}$ of cardinality κ with countable $\varphi(\mathfrak{M})$. Use this to show that if a countable complete theory is categorical in an uncountable cardinality, then it has no Vaughtian pairs. (Use Lachlan's Theorem.)

Aufgabe 3. Show directly (without using that ω-stability implies the existence of saturated models in all infinite cardinalities) that a countable theory T which is categorical in some uncountable cardinality, has a model $\mathfrak{M}$ of cardinality $\aleph_1$ in which each $L(M)$-formula is either satisfied by a finitely many or by $\aleph_1$ many elements.

Aufgabe 4. Show that the theory RG of the random graph has a Vaughtian pair.

Aufgabe 5. Let T be a theory, $\mathfrak{M}$ a model of T and $\overline{a} \subseteq M$ a finite tuple of parameters. Let $q(\overline{x})$ be the type of $\overline{a}$ in $\mathfrak{M}$. Then, for new constants $\overline{c}$, the $L(\overline{c})$-theory

$$T(q) = \mathrm{Th}(\mathfrak{M}, \overline{a}) = T \cup \{\varphi(\overline{c}) |\ \varphi(\overline{x}) \in q(\overline{x})\}$$

is complete. Prove the following:

1. T is λ-stable if and only if $T(q)$ is,
2. T has a Vaughtian pair if and only if $T(q)$ has one,
3. T is κ-categorical if and only if $T(q)$ is.

[0] http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/