Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 2.11.2012
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Blatt 2

Aufgabe 1. Let L be a language and P be a new n-ary relation symbol. Let $\varphi = \varphi(P)$ be an $L(P) = L \cup \{P\}$-sentence and $\pi(x_1, \ldots, x_n)$ an L-formula. Now replace every occurrence of P in φ by π. More precisely, every subformula of the form $Pt_1 \ldots t_n$ is replaced by $\pi(t_1 \ldots t_n)$. We denote the resulting L-formula by $\varphi(\pi)$. Show that

$$\mathfrak{A} \models \varphi(\pi) \text{ if and only if } (\mathfrak{A}, \pi(\mathfrak{A})) \models \varphi(P).$$

Aufgabe 2.

1. For a prime number p, let $\mathbb{Z}_{p^\infty}$ denote the p-Prüfer group, i.e., the group of all p^k-th roots of unity for all $k \in \mathbb{N}$. Show that the groups $\mathbb{Z}_{p^\infty}^m$ and $\mathbb{Z}_{p^\infty}^n$ are not elementarily equivalent for $m \neq n$.
2. Show that $\mathbb{Z}^n \not\equiv \mathbb{Z}^m$ in the language of abelian groups if $n \neq m$.

Aufgabe 3. Show that if $\mathfrak{A}$ is a finite L-structure and $\mathfrak{B}$ is elementarily equivalent to $\mathfrak{A}$, then they are isomorphic. (Show this first for finite L.)

Aufgabe 4. Use ultraproducts to show that the class of all finite groups (all torsion groups, all nilpotent groups, respectively) does not form an elementary class.

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[0] http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/