Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 16.11.2012
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Blatt 4

Aufgabe 1. Let $\mathfrak{A}$ be an L-structure and $(\mathfrak{A}_i)_{i\in I}$ a chain of elementary substructures of $\mathfrak{A}$. Show that $\bigcup_{i\in I}\mathfrak{A}_i$ is an elementary substructure of $\mathfrak{A}$.

Aufgabe 2. A class $\mathcal{C}$ of L-structures is *finitely axiomatisable* if it is the class of models of a finite theory. Show that $\mathcal{C}$ is finitely axiomatisable if and only if both $\mathcal{C}$ and its complement form an elementary class.

Aufgabe 3. Show that the class of connected graphs is not an elementary class. A *graph* (V, R) is a set V with a symmetric, irreflexive binary relation R. It is *connected* if for any $x, y \in V$ there is $n \in \mathbb{N}$ and a sequence of elements $x_0 = x, \ldots, x_n = y$ such that $(x_{i-1}, x_i) \in R$ for $i = 1, \ldots, n$.

Aufgabe 4. Let T be an $\mathrm{L}_{\mathrm{Ring}}$-theory containing Field. Show that:

1. If T has models of arbitrary large characteristic, then it has a model of characteristic 0.
2. The theory of fields of characteristic 0 is not finitely axiomatisable.

Aufgabe 5 (freiwillig)**.** Find an example of structures $\mathfrak{A}$ and $\mathfrak{B}$ such that $\mathfrak{A}$ is a substructure of $\mathfrak{B}$ and $\mathfrak{A}$ and $\mathfrak{B}$ are elementarily equivalent, but $\mathfrak{A}$ is not an elementary substructure of $\mathfrak{B}$.

[0] http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/