Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 30.11.2012
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Blatt 6

Aufgabe 1. Let X be a topological space, Y_1 and Y_2 quasi-compact[1] subsets, and $\mathcal{H}$ a set of clopen subsets. Then the following are equivalent:

a) There is a positive Boolean combination B of elements from $\mathcal{H}$ such that $Y_1 \subseteq B$ and $Y_2 \cap B = \emptyset$.

b) For all $y_1 \in Y_1$ and $y_2 \in Y_2$ there is an $H \in \mathcal{H}$ such that $y_1 \in H$ and $y_2 \notin H$.

(This, in fact, is a generalisation of the Separation Lemma.)

Aufgabe 2. A theory T with quantifier elimination is axiomatisable by sentences of the form

$$\forall x_1 \dots x_n \psi$$

where ψ is a primitive existential formula.

Aufgabe 3. Let Graph be the theory of graphs. The theory RG of the *random graph* is the extension of Graph by the following axiom scheme:

$$\forall x_0 \dots x_{m-1}\, y_1 \dots y_{n-1} \Big(\bigwedge_{i \neq j} \neg x_i \doteq y_j \to \exists z \big(\bigwedge_{i<m} zRx_i \wedge \bigwedge_{j<n} (\neg zRy_j \wedge \neg z \doteq y_j) \big) \Big)$$

Show that RG has quantifier elimination and is complete. Show also that RG is the model companion of Graph.

Hint: Proceed as in the case of dense linear orders.

Aufgabe 4. Show that the following is true in any algebraically closed field K: every injective polynomial map of a definable subset of K^n in itself is surjective.

"Hint": Here you can use the fact that the theory of algebraically closed fields has quantifier elimination (which will be proved in section 3.3).

[1] That is, compact but not necessarily Hausdorff.

[1] `http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/`