Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 7.12.2012
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Blatt 7

Aufgabe 1. Let T be the theory of nontrivial divisible torsion-free abelian groups in the language of abelian groups. Show that $T_\forall$ is the theory of torsion-free abelian groups.

Aufgabe 2. Consider the theory of $(\mathbb{Z}, +, 0, 1)$ in the language where we add predicates P_n for the elements divisible by n, for all $n \geq 2$. Axiomatize this theory and show that it has quantifier elimination.

Hint: Besides the axioms for abelian groups, consider sentences expressing, for a structure $(G, +, 0, 1, (P_n^G)_n)$, the conditions $P_n^G = nG$ and $G/P_n^G \cong \mathbb{Z}/n\mathbb{Z}$.

Aufgabe 3. Let $A_i, i \leq k$, be any sets. Show that if A_0 is finite, then: $A_0 \subseteq \bigcup_{i=1}^k A_i$ if and only if

$$\sum_{\Delta \subseteq \{1,\dots,k\}} (-1)^{|\Delta|} \Big| A_0 \cap \bigcap_{i \in \Delta} A_i \Big| = 0.$$

Aufgabe 4 (B. Neumann)**.**

1. Let G be a group (not necessarily abelian), and $H_0, \dots, H_n$, subgroups of infinite index. Show that G is not a finite union of cosets of the H_i.

   Hint: Proceed by induction on n. For $n > 0$, distinguish two cases depending on whether for some $i < n$, $H_n \cap H_i$ has finite index in H_n.

2. Let H_i denote subgroups of some abelian group. Prove: if $H_0 + a_0 \subseteq \bigcup_{i=1}^n H_i + a_i$ and $H_0/(H_0 \cap H_i)$ is infinite for $i > k$, then $H_0 + a_0 \subseteq \bigcup_{i=1}^k H_i + a_i$.

   Hint: Apply part 1.

[0] http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/