Übungen zur Vorlesung **Modelltheorie** (WS 2012/13) 21.12.2012
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Blatt 9

Aufgabe 1. Show that a type p is isolated if and only if it is isolated as an element in the Stone space.

Aufgabe 2. Prove the following:

(a) Closed subsets of $\mathrm{S}_n(T)$ have the form $\{p \in \mathrm{S}_n(T) \mid \Sigma \subseteq p\}$, where Σ is any set of formulas.

(b) Let T be countable and consistent. Then any meagre[1] subset X of $\mathrm{S}_n(T)$ can be omitted, i.e., there is model which omits all $p \in X$.

Aufgabe 3. Let B be a subset of $\mathfrak{A}$. Show that the *restriction (of variables)* map $\mathrm{S}_{n+m}(B) \to \mathrm{S}_n(B)$ given by $\mathrm{tp}(a, c/B) \mapsto \mathrm{tp}(a/B)$ is open, continuous and surjective. Let a be an n-tuple in A. Show that the fibre over $\mathrm{tp}(a/B)$ is canonically homeomorphic to $\mathrm{S}_m(aB)$.

Aufgabe 4. Consider the structure $\mathfrak{M} = (\mathbb{Q}, <)$. Determine all types in $\mathrm{S}_1(\mathbb{Q})$. Which of these types are realised in $\mathbb{R}$? Which extensions does a type over $\mathbb{Q}$ have to a type over $\mathbb{R}$?

[1] A subset of a topological space is *nowhere dense* if its closure has no interior. A countable union of nowhere dense sets is *meagre*.

[1] `http://home.mathematik.uni-freiburg.de/caycedo/lehre/ws12_modell/`