Exercises for the lecture *Model Theory of Fields* (WS 15/16) 7.11.2015

Week 4

1. (Ax-Grothendieck theorem) Let K be an algebraically closed field and $f : K^n \to K^n$ be a polynomial map. Show that if f is injective then f is also surjective.

Hint: Prove it first for $K = \mathbb{F}_p^{\mathrm{alg}}$ using the analogous property of finite fields.

2. Let K be an algebraically closed field and let $c_1, \ldots, c_m \in K$. Find a tuple $d \subset K$ such that for any automorphism σ of K we have: σ restricts to a permutation of the set $\{c_1, \ldots, c_m\}$ if and only if σ fixes each coordinate of d.

Generalise the above to all $c_1, \ldots, c_m \in K^n$.