

**Week 5**

**1.** Let  $K$  be a field and  $W \subset K^n$  be a  $K$ -linear subspace. Notice that  $W$  is definable in the field structure, over  $K$ . Show that there is a canonical parameter for  $W$  in  $K^m$ , for some  $m$ .

*Hint:* Let  $e_1, \dots, e_n$  be the standard basis of  $K^n$ . Let  $e_{i_1}, \dots, e_{i_k}$  be such that  $e_{i_1} + W, \dots, e_{i_k} + W$  is a basis of the quotient  $K^n/W$ . For  $i = 1, \dots, n$ , write:

$$e_i = \sum_{j=1}^k \beta_{ij} e_{i_j} + w_i$$

with each  $\beta_{ij}$  in  $K$  and each  $w_i$  in  $W$ . Then  $\beta := (\beta_{ij}) \in K^{nk}$  is a canonical parameter for  $W$ , i.e. for any  $\sigma \in \text{Aut}(K)$ ,  $\sigma(\beta) = \beta$  if and only if  $\sigma[W] = W$ .

**2.** Let  $K$  be a field. Show that, similarly to 1. above, for every  $K$ -linear subspace  $W$  of  $\bigoplus_{i < \omega} K$ , there is  $\beta \in K^\omega$  which works as a canonical parameter for  $W$ , i.e. for any  $\sigma \in \text{Aut}(K)$ ,  $\sigma(\beta) = \beta$  if and only if  $\sigma[W] = W$ .