Week 9

1. Let (K, d) be a differential field and $V \subset K^n$ an irreducible affine algebraic variety defined over K, by the ideal $I(V)$.

The tangent space $T_a(V)$ of V at a is defined by the equations $f'(a) \cdot y = 0$, for all $f \in I(V)$. The tangent bundle $T(V)$ of V is defined by the equations $f(x) = 0$ and $f'(x) \cdot y = 0$, for all $f \in I(V)$.

Notice that if V is defined over the field of constants of K, then the torsor $\mathcal{T}(V)$ of V coincides with $T(V)$.

In general, explain how for each $a \in V$, the set $\mathcal{T}_a(V) := \{y : (a, y) \in \mathcal{T}(V)\}$ is a *torsor* (a.k.a. *principal homogeneous space*) for $T_a(V)$.

2. Show that if (K, d) is a model of DCF_0, then its field of constants is algebraically closed.

3. Is DCF_0 finitely axiomatizable?