Abteilung für Mathematische Logik
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Infinite Games

Sommersemester 2019

Exercise sheet 2, 09.05.2019

1. Prove Claim 2 of the lecture. Let ρ be a strategy for player II and $t \in \omega^{<\omega}$ so that ρ in not a winning strategy for II in $G_{<\infty}(A_I, A_{II}, (\rho * t))$. Then there exists x_0 such that for all y_0 player II still does not have a winning strategy in $G_{<\infty}(A_I, A_{II}, (\rho * t)^\frown\langle x_0, y_0\rangle)$.

2. Let $G_{<\infty}(A_I, A_{II})$ be a finite-unbounded game and σ a winning strategy for I. We introduce the following notation:

   - A position $\sigma * t$ of the game is a *win-in-n* for I if, assuming player I follows σ in the game $G_{<\infty}(A_I, A_{II}, (\rho * t))$, he will win in at most n moves.
   - Given the first move x_0 of player I, we say that the move y_0 of player II is *non-optimal* if there is $n \in \omega$ such that $\langle x_0, y_0\rangle$ is a *win-in-n* position for player I (otherwise we say y_0 is optimal).

   Assume σ is a winning strategy for I, player II only has finitely many *legal moves* (i.e., moves for which player II does not lose the game) and the initial position is not a *win-in-n* strategy for player I (for any n). Prove that there is at least one player II's move y_0 which is optimal.

3. Use Exercise 2 to show the following: if $G_{<\infty}(A_I, A_{II})$ is a finite-unbounded game, σ is a winning strategy for I, and at every move, player II only has finitely many legal moves to play, then there exists $N \in \omega$ such that, if player I follows the strategy σ, he will win in at most N moves (no matter what player II plays).