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Infinite Games

Sommersemester 2019

Exercise sheet 3, 23.05.2019

1. Let A be a countable set. Show that $G(A)$ is determined (in particular, show that II has a winning strategy).

2. Consider the game $G'(A)$ with the following rules: I and II pick numbers x_i and y_i. Let z be the result of the infinite game. Player I wins iff the infinite sum $\sum_{i=0}^{\infty} \frac{1}{x_i+1} + \frac{1}{y_i+1}$ is convergent, i.e.,

$$A := \{z \in \omega^\omega : \sum_{i=1}^{\infty} \frac{1}{z(n)+1} < \infty\}.$$

   Is $G'(A)$ determined? in case yes, who has a winning strategy?

   Then consider the game $G''(A)$ as above but with the additional rule that player II's move must be at least as large as the preceding move by player I, i.e.,

$$A := \{z \in \omega^\omega : \forall n(z(2n+1) \geq z(2n)) \wedge \sum_{i=1}^{\infty} \frac{1}{z(n)+1} < \infty\}.$$

   Is $G''(A)$ determined? in case yes, who has a winning strategy?

3. Consider the following definition: $A \subseteq 2^\omega$ has the *Baire property* iff there is an open set O such that $A \triangle O$ is meager.

   Show that any Borel set has the Baire property.

   (*Hint*: use recursive induction on the Borel hierarchy.)