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Infinite Games

Sommersemester 2019

Exercise sheet 6, 12.07.2019

1. Let $X := Y^\omega$ (for some set of utilities Y) and $R \subseteq X \times X$ a reflexive and transitive social welfare relation satisfying Pareto and finite anonimity. Show that the existence of an ultrafilter on ω $\mathfrak{U} \supseteq \mathfrak{F} := \{a \subseteq \omega : a \text{ is finite}\}$ implies the existence of $R \subseteq X \times X$ reflexive, transitive and total social welfare relation satisfying Pareto and finite anonimity.

2. Let $\mathfrak{U}_\sigma, \mathfrak{U}'_\sigma, \mathfrak{U}''_\sigma$ as defined in the lecture. Show that they are all non-principal filters.

3. Find a total, reflexive and transitive relation R on some set X that cannot be represented by a continuous social welfare function, i.e, there is no continuous function $f : X \to [0, 1]$ such that $xRy \Leftrightarrow f(x) < f(y)$. (*Hint*: consider $X = \mathbb{R} \times \mathbb{R}$ and R the lexicografic order)