1^{st} Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

Giorgio Laguzzi

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Exercise 1

Given $s \in \omega^{<\omega}$ we define

if k is even, then $a_n^k := \langle 1, 1, , \dots, 1, 1, 0 \rangle$, with $|a_n^k| = n + 1$

if k is odd, then $a_n^k := \langle 0, 0, \dots, 0, 0, 1 \rangle$, with $|a_n^k| = n + 1$

For every $x \in \omega^\omega$, let $j(x) := a^0_{x(0)}{}^\frown a^1_{x(1)}{}^\frown \dots {}^\frown a^n_{x(n)}{}^\frown \dots$.

Let $C := \{x \in 2^\omega : \forall n \exists m_0, m_1 > n(x(m_0) \neq x(m_1))\}$, i.e., C consists of those binary sequences which are not eventually constant.

Show that $j : \omega^\omega \to 2^\omega \setminus C$ is a homeomorphism.

Exercise 2

Let $k : 2^\omega \to [0, 1]$ be the *binary expansion*, i.e., for $x \in 2^\omega$,

$$k(x) := \sum_{n \in \omega} x(n) \cdot 2^{-n+1},$$

and let $D := 2^\omega \setminus C$ (C as in the previous exercise).

Show that k is continuous and that $k{\restriction}D$ is a homeomorphism.