

# 11<sup>th</sup> Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

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## Exercise 21

Let  $\mathbb{P}_\kappa := (\mathbb{P}_\alpha, \mathbb{Q}_\alpha : \alpha < \kappa)$  be a  $\kappa$ -iteration with  $I$ -support, where  $I \subseteq \mathcal{P}(\kappa)$  is an ideal on  $\kappa$ , such that each member of  $I$  is bounded in  $\kappa$ . Let  $G$  be  $\mathbb{P}_\kappa$ -generic over  $V$ ,  $S \in V$  such that  $V[G] \models |S| < \text{cf}(\kappa)$  and  $X \subseteq S$  in  $V[G]$ . Then there exists  $\alpha < \kappa$  such that  $X \in V[G \restriction \alpha]$ .

## Exercise 22

Let  $\kappa$  be a regular uncountable cardinal and  $\delta$  be any limit ordinal. Let  $\mathbb{P}_\delta$  be an iteration such that for all  $\alpha < \delta$ ,  $\mathbb{P}_\delta \restriction \alpha$  satisfies the  $\kappa$ -cc. If  $\mathbb{P}_\delta$  is a direct limit and either  $\text{cf}(\delta) \neq \kappa$  or there are stationary many  $\alpha$ 's such that  $\mathbb{P}_\delta \restriction \alpha$  is a direct limit, then  $\mathbb{P}_\delta$  satisfies the  $\kappa$ -cc.