

12th Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

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Exercise 23

Let $i : \mathbb{P} \rightarrow \mathbb{Q}$ be a dense embedding. Define, by recursion on $\dot{x} \in V^{\mathbb{P}}$,

$$i'(\dot{x}) := \{(i'(\dot{y}), i(p)) : (\dot{y}, p) \in \dot{x}\}.$$

Show that, for any formula $\varphi(v_1, \dots, v_n)$, one has

$$p \Vdash \varphi(\dot{x}_1, \dots, \dot{x}_n) \text{ iff } i(p) \Vdash \varphi(i'(\dot{x}_1), \dots, i'(\dot{x}_n)).$$

(Hint: see Lemma 7.13 on page 222 in Kunen's book.)

Exercise 24

Let κ be inaccessible and \mathbb{P} be a forcing notion with $|\mathbb{P}| < \kappa$. Show that $\Vdash_{\mathbb{P}}$ “ κ is inaccessible”.

(Hint: see Proposition 10.12 on page 125 in Kanamori's book.)