14^{th} Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

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Exercise 26

Let Γ be a topologically reasonable family. Show that the following implications hold in ZFC:

1. $\Gamma(\mathbb{C}) \Rightarrow \Gamma(\mathbb{V})$
2. $\Gamma(\mathbb{M}) \Rightarrow \Gamma(\mathbb{S})$
3. $\Gamma(\mathbb{B}) \Rightarrow \Gamma(\mathbb{V})$

Exercise 27

Show that $\mathbf{\Sigma}^1_2(\mathbb{V}) \not\Rightarrow \mathbf{\Delta}^1_2(\mathbb{L})$.

(Hint: consider the ω_1-iteration of Cohen forcing with countable support. Show that in such an extension $\mathbf{\Sigma}^1_2(\mathbb{V})$ holds, while no dominating reals are added. Btw, in such an extension not only $\mathbf{\Sigma}^1_2(\mathbb{V})$ holds but even $\textsc{Proj}(\mathbb{V})$ ...)