

5th Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

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Exercise 9

Let M be a c.t.m. of ZFC, $\mathbb{P} \in M$ and G \mathbb{P} -generic filter over M , and $E \subseteq \mathbb{P}$ be dense *below* p (i.e., $\forall q \leq p \exists q' \leq q (q' \in E)$.) Show the following facts.

1. $\forall \tau \in M^{\mathbb{P}} (\text{rank}(\tau_G) \leq \text{rank}(\tau))$.
2. $\text{On} \cap M[G] = \text{On} \cap M$ (where On is the class of all ordinals.)
3. if $p \in G$, then $G \cap E \neq \emptyset$.

Exercise 10

Show that ccc forcings preserves cardinals.

(Hint: see Chapter 7, Theorem 5.10 in [Ku80].)