9^{th} Exercise Sheet, Set Theory of the Real Line, WS 2014/2015

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Exercise 17

Let $\mathbb{P} \lessdot \mathbb{Q}$ (i.e., $\mathbb{P}$ complete embeds in $\mathbb{Q}$). Show that $\mathbb{P}$ and $\mathbb{P} * (\mathbb{P} : \mathbb{Q})$ are forcing equivalent.

(Hint: see Lemma 1.4.28 in [BJ95].)

Exercise 18

Let $\{\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha : \alpha < \delta\}$ be a finite/countable support iteration of proper forcing. Let $\alpha + \beta = \delta$. Then there exists a $\mathbb{P}_\alpha$-name $\{\mathbb{P}'_\gamma, \dot{\mathbb{Q}}'_\gamma : \gamma < \beta\}$ for a finite/countable support iteration of length β such that

$$\Vdash_\alpha \mathbb{P}'_\gamma \text{ is equivalent to } (\mathbb{P}_\delta : \mathbb{P}_\alpha).$$

(Hint: see Theorem 1.5.10 in [BJ95].)