Abteilung für Mathematische Logik
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Infinitary Combinatorics
Wintersemester 2019/20

Übungsblatt 10

Abgabe am 13.2.2020 vor der Vorlesung

Remind the following types of infinite trees.

Given a tree p remind that stem(p) is the longest node in p compatible with all other nodes in p.

- $p \subseteq 2^{<\omega}$ is called a *perfect* tree iff for every $s \in p$ there exists $t \in p$ such that $t \supseteq s$ and $t^\frown 0, t^\frown 1 \in p$ (such t is called a *splitting node*).
- $p \subseteq 2^{<\omega}$ is *Silver* iff p is perfect and for every $s, t \in p$, with $|s| = |t|$ one has $s^\frown 0 \in p \Leftrightarrow t^\frown 0 \in p$ and $s^\frown 1 \in p \Leftrightarrow t^\frown 1 \in p$.
- $p \subseteq \omega^{<\omega}$ is called *Miller* tree iff p is perfect and for every $s \in p$ there exists $t \supseteq s$ in p such that there are infinitely many distinct $n_k \in \omega$ such that for all $k \in \omega$, $t^\frown n_k \in p$.
- $p \subseteq \omega^{<\omega}$ is called *Laver* tree iff p is perfect and for every $t \supseteq p$ extending stem(p) there are infinitely many distinct $n_k \in \omega$ such that for all $k \in \omega$, $t^\frown n_k \in T$.

N.B.: In all questions in the following three exercises please provide an argument for either a positive or a negative answer.

Exercise 1. Let X be any comeager set.

- (2 points) is there a perfect tree p such that $[p] \subseteq X$?
- (3 points) is there a Silver tree p such that $[p] \subseteq X$?
- (3 points) is there a Miller tree p such that $[p] \subseteq X$?
- (4 points) is there a Laver tree p such that $[p] \subseteq X$?

Remind the Lebsgue measure μ on 2^ω and ω^ω be defined as follows:

- let m be the measure on $\{0,1\}$ such that $m(\{0\}) = m(\{1\}) = \frac{1}{2}$ and then put μ be the corresponding product measure on 2^ω.
- let m be the measure on ω such that $m(\{n\}) = \frac{1}{n}$ and then put μ be the corresponding product measure on ω^ω.

Exercise 2. Let X be any set with $\mu(X) = 1$.

- (2 points) is there a perfect tree p such that $[p] \subseteq X$?
- (3 points) is there a Silver tree p such that $[p] \subseteq X$?
- (3 points) is there a Miller tree p such that $[p] \subseteq X$?
- (4 points) is there a Laver tree p such that $[p] \subseteq X$?

Let δ be the so called *dominating topology* on ω^ω, i.e. the topology generated by the following basic open sets:

$$[t, f] := \{x \in \omega^\omega : x \supset t \wedge \forall n \in \omega (f(n) \leq x(n))\}$$

where $t \in \omega^{<\omega}$, $f \in \omega^\omega$ with $f \supset t$.

Exercise 3. Let X be any set comeager w.r.t δ.

- (2 points) is there a perfect tree p such that $[p] \subseteq X$?
- (3 points) is there a Silver tree p such that $[p] \subseteq X$?
- (3 points) is there a Miller tree p such that $[p] \subseteq X$?
- (4 points) is there a Laver tree p such that $[p] \subseteq X$?