Abteilung für Mathematische Logik
Dr. Giorgio Laguzzi
Christian Bräuninger

Infinitary Combinatorics
Wintersemester 2019/20

Übungsblatt 6

Abgabe am 10.12.2019 vor der Vorlesung

Exercise 1. (4 points) Let $S \subset \kappa$ be stationary and $f : S \to \kappa$ funtion so that $\forall \alpha < \kappa (f(\alpha) < \alpha)$. Then there exists $S' \subseteq S$ stationary such that $f \restriction S'$ is constant, i.e., there is $S' \subseteq S$ stationary such that $\exists \gamma < \kappa \forall \alpha \in S' (f(\alpha) = \gamma)$.

Exercise 2. (4 points) Consider ω_1 with the order topology and let $f : \omega_1 \to \mathbb{R}$ be a continuous function. Show that

$$\exists \alpha < \omega_1 \forall \beta > \alpha (f(\beta) = f(\alpha)).$$

Exercise 3. (4 points)

Let κ be a regular cardinal and $\mathfrak{B} := \mathfrak{P}(\kappa)/\mathrm{Cub}_\kappa^*$. Denote by $[A]$ the equivalence class of $A \subseteq \kappa$.

- Show that $\mathfrak{B}$ is a Boolean algebra
- Given $\{A_\alpha : \alpha < \kappa\}$ familiy of subsets of κ, show that $\bigwedge_{\alpha<\kappa}[A_\alpha]$ always exists and it is equal to $[D]$, where D is the diagonal intersection

$$D := \{\gamma < \kappa : \forall \alpha < \gamma (\gamma \in C_\alpha).$$