Abteilung für Mathematische Logik
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Infinitary Combinatorics
Wintersemester 2019/20

Übungsblatt 8

Abgabe am 7.1.2020 vor der Vorlesung

Exercise 1. (8 points) $\diamondsuit^+$ implies that there exists a family $F \subseteq P(\omega_1)$ such that:

1. $\forall \beta < \omega_1 (|\{x \cap \beta : x \in F\}| \leq \omega)$, and
2. $\forall A \subseteq \omega_1 (|A| = \omega_1 \Rightarrow \exists x \in F(|x| = \omega_1 \wedge x \subseteq A))$.

Hint: Let $\{\mathcal{A}_\alpha : \alpha < \omega_1\}$ be a $\diamondsuit^+$-sequence. Given $C \subseteq \omega_1$ and $\xi < \omega_1$, let $s(C, \xi) := \sup(C \cup \{0\}) \cap (\xi + 1)$ and for $A \subseteq \omega_1$, let $X(A, C) := \{\xi \in A : \neg \exists \eta \in A(s(C, \xi) \leq \eta < \xi)\}$. Then consider F be the family of those $X(A, C)$ satisfying:

- $\forall \alpha \in C(A \cap \alpha \in \mathcal{A}_\alpha)$
- $\forall \alpha \in C(C \cap \alpha \in \mathcal{A}_\alpha)$
- $|A| = \omega_1$ and C club.

Exercise 2. (4 points) $\diamondsuit^+$ implies that there is an ω_1-Kurepa tree with 2^{ω_1} many paths.

Hint: Use properties (1) and (2) from exercise 1.