Abteilung für Mathematische Logik
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Infinitary Combinatorics
Wintersemester 2019/20

Übungsblatt 9

Abgabe am 21.1.2020 vor der Vorlesung

Exercise 1. (3 points) Given a σ-ideal I on X such that for every $x \in X$, $\{x\} \in I$. Show that $\mathsf{add}(I) \leq \mathsf{cov}(I), \mathsf{non}(I) \leq \mathsf{cof}(I)$.

Exercise 2. (3 points) Assume $I \leq_T J$, for two given ideals I, J. Show that $\mathsf{add}(I) \geq \mathsf{add}(J)$ and $\mathsf{cof}(I) \leq \mathsf{cof}(J)$.

Exercise 3 (3 points) Let $\mathbb{P} := \{p \subseteq 2^\omega : \mu([p]) > 0 \wedge p \text{ is a perfect tree}\}$, ordered by inclusion $q \leq p \Leftrightarrow q \subseteq p$. Show that $\mathbb{P}$ satisfies ccc.

Exercise 4 (3 points) Given a sequence $\{k_n : n \in \omega\}$ of real numbers in $(0, 1)$ show that there is a family $\{U_n : n \in \omega\}$ consisting of open subsets of 2^ω such that $\mu(U_n) = k_n$ and for every $n, m \in \omega$, $n \neq m$, one has $\mu(U_n \cap U_m) = \mu(U_n) \cdot \mu(U_m)$.