**Abteilung für Mathematische Logik**
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**Infinitary Combinatorics**
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# Übungsblatt 1

Abgabe am 29.10.2019 vor der Vorlesung

**Excercise 1.** Show that there is a family $\mathcal{A}$ of $\omega_\omega$ many finite sets such that no $\mathcal{B} \subseteq \mathcal{A}$ of cardinality $\omega_\omega$ forms a $\Delta$-system.

**Excercise 2.** Show that if $\kappa \leq 2^\omega$ and $X_\alpha$ are separable spaces for $\alpha < \kappa$, then $\prod_{\alpha<\kappa} X_\alpha$ is separable.

*Hint:* Consider first the space ${}^I X$, where $I \subseteq {}^\omega 2$ and $X$ is separable. Let $D$ be dense in $X$. Let $E$ be the set of $\varphi \in {}^I D$, such that for some $n \in \omega$,

$$\forall f, g \in I \, (f \restriction n = g \restriction n \to \varphi(f) = \varphi(g)) \, .$$

Then $E$ is dense in $X$.

**Excercise 3.** Show that if $\kappa > 2^\omega$, then the space ${}^\kappa 2$ (where $2 = \{0, 1\}$ has the discrete topology) is not separable.

*Hint:* If $D \subseteq {}^\kappa 2$ is countable, show that there are $\alpha < \beta$ such that $\forall f \in D \, (f(\alpha) = f(\beta))$.