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Infinitary Combinatorics
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Übungsblatt 2

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For $x, y \in \omega^\omega$ we say that y *eventually dominates* x ($x \leq^* y$) if there exists $n \in \omega$ such that for all $k \geq n$ we have $x(k) \leq y(k)$.

A family $F \subseteq \omega^\omega$ is called *dominating* if for all $x \in \omega^\omega$ there is $y \in F$ such that $x \leq^* y$, and *unbounded* if for all $x \in \omega^\omega$ there is $y \in F$ such that $y \not\leq^* x$.

Exercise 1. Assume $\mathsf{MA}(\kappa)$ for $\omega < \kappa < 2^\omega$.
Show that there is no dominating family of size κ.

Exercise 2. Assume $\mathsf{MA}(\kappa)$ for $\omega < \kappa < 2^\omega$.
Show that there is no undbounded family of size κ.

For $s \in [\omega]^{<\omega}$, $A \in [\omega]^\omega$ consider the set

$$[s, A] := \{X \in [\omega]^\omega \mid s \subseteq X \subseteq A\}.$$

The *Ellentuck topology* τ_E on $[\omega]^\omega$ is the topology generated by these sets $[s, A]$.
Recall that $N \subseteq [\omega]^\omega$ is called τ_E-closed nowhere dense if for all $s \in [\omega]^{<\omega}$, $A \in [\omega]^\omega$ there are $s' \in [\omega]^{<\omega}$, $A' \in [\omega]^\omega$ such that $[s', A'] \subseteq [s, A]$ and $[s', A'] \cap N = \emptyset$.

Exercise 3. Assume $\mathsf{MA}(\kappa)$ for $\omega < \kappa < 2^\omega$.
Show that for every κ-sized family F of τ_E-closed nowhere dense subsets of $[\omega]^\omega$ we have $\bigcup F \subsetneq [\omega]^\omega$.