Abteilung für Mathematische Logik
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Infinitary Combinatorics
Wintersemester 2019/20

Übungsblatt 4

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Exercise 1. (4 points) Show that a complete Boolean algebra $\mathcal{B}$ has the countable chain condition (c.c.c.) if and only if there is no uncountable sequence $\langle b_\alpha : \alpha < \omega_1 \rangle$ in $\mathcal{B}$ such that $\alpha < \beta \to b_\alpha < b_\beta$.

Exercise 2. (4 points) Show that the product space 2^{ω_1} is an example of a compact c.c.c. Hausdorff space in which (regardless of the axioms of set theory) there is a union of ω_1 many closed nowhere dense sets which is not of first category, i.e. can not be written as a countable union of closed nowhere dense sets.

$\langle \mathbb{P}, \leq \rangle$ is called *separative* if $\leq$ is a partial order in the strict sense and whenever $p \not\leq q$, there is an r such that $r \leq p$ and $r \perp q$.

Recall that for a partial order $\mathbb{P}$ there is a complete Boolean algebra $\mathcal{B}$ and a map $i : \mathbb{P} \to \mathcal{B} \setminus \{\emptyset\}$ such that:

1. $i[\mathbb{P}]$ is dense in $\mathcal{B} \setminus \{\emptyset\}$.
2. $\forall p, q \in \mathbb{P}\,(p \leq q \to i(p) \leq i(q))$.
3. $\forall p, q \in \mathbb{P}\,(p \perp q \leftrightarrow i(p) \cap i(q) = \emptyset)$.

Exercise 3. (4 points) Show that $\mathbb{P}$ is separative if and only if i is injective and satisfies

$$\forall p, q \in \mathbb{P}\,(p \leq q \leftrightarrow i(p) \leq i(q))\,.$$

Bonus. Give an example of a partial order $\mathbb{P}$ such that i is injective, but $\mathbb{P}$ is not separative.