

Contents

Preface	v
PART A	1
I Introduction	3
§1 An Example from Group Theory	4
§2 An Example from the Theory of Equivalence Relations	5
§3 A Preliminary Analysis	6
§4 Preview	8
II Syntax of First-Order Languages	11
§1 Alphabets	11
§2 The Alphabet of a First-Order Language	13
§3 Terms and Formulas in First-Order Languages	15
§4 Induction in the Calculus of Terms and in the Calculus of Formulas	19
§5 Free Variables and Sentences	24
III Semantics of First-Order Languages	27
§1 Structures and Interpretations	28
§2 Standardization of Connectives	31
§3 The Satisfaction Relation	32
§4 The Consequence Relation	33
§5 Two Lemmas on the Satisfaction Relation	40
§6 Some Simple Formalizations	44
§7 Some Remarks on Formalizability	48
§8 Substitution	52
IV A Sequent Calculus	59
§1 Sequent Rules	60
§2 Structural Rules and Connective Rules	62
§3 Derivable Connective Rules	63
§4 Quantifier and Equality Rules	66
§5 Further Derivable Rules and Sequents	68
§6 Summary and Example	69
§7 Consistency	72

V The Completeness Theorem	75
§1 Henkin's Theorem	75
§2 Satisfiability of Consistent Sets of Formulas (The Countable Case)	79
§3 Satisfiability of Consistent Sets of Formulas (The General Case)	82
§4 The Completeness Theorem	85
VI The Löwenheim-Skolem and the Compactness Theorem	87
§1 The Löwenheim-Skolem Theorem	87
§2 The Compactness Theorem	88
§3 Elementary Classes	91
§4 Elementarily Equivalent Structures	94
VII The Scope of First-Order Logic	99
§1 The Notion of Formal Proof	99
§2 Mathematics within the Framework of First-Order Logic	103
§3 The Zermelo-Fraenkel Axioms for Set Theory	107
§4 Set Theory as a Basis for Mathematics	110
VIII Syntactic Interpretations and Normal Forms	115
§1 Term-Reduced Formulas and Relational Symbol Sets	115
§2 Syntactic Interpretations	118
§3 Extensions by Definitions	125
§4 Normal Forms	128
PART B	135
IX Extensions of First-Order Logic	137
§1 Second-Order Logic	138
§2 The System $\mathcal{L}_{\omega_1\omega}$	142
§3 The System \mathcal{L}_Q	148
X Limitations of the Formal Method	151
§1 Decidability and Enumerability	152
§2 Register Machines	157
§3 The Halting Problem for Register Machines	163
§4 The Undecidability of First-Order Logic	167

§5	Trahtenbrot's Theorem and the Incompleteness of Second-Order Logic	170
§6	Theories and Decidability	173
§7	Self-Referential Statements and Gödel's Incompleteness Theorems	181
XI	Free Models and Logic Programming	189
§1	Herbrand's Theorem	189
§2	Free Models and Universal Horn Formulas	193
§3	Herbrand Structures	198
§4	Propositional Logic	200
§5	Propositional Resolution	207
§6	First-Order Resolution (without Unification)	218
§7	Logic Programming	226
XII	An Algebraic Characterization of Elementary Equivalence	243
§1	Finite and Partial Isomorphisms	244
§2	Fraïssé's Theorem	249
§3	Proof of Fraïssé's Theorem	251
§4	Ehrenfeucht Games	258
XIII	Lindström's Theorems	261
§1	Logical Systems	261
§2	Compact Regular Logical Systems	264
§3	Lindström's First Theorem	266
§4	Lindström's Second Theorem	272
	References	277
	Symbol Index	280
	Subject Index	283