

---

## Contents

<b>1</b>	<b>Berlin 1871–1897</b> . . . . .	1
1.1	Family and Youth . . . . .	1
1.2	School and University Studies . . . . .	5
1.3	Ph. D. Thesis and the Calculus of Variations . . . . .	10
1.4	The Boltzmann Controversy . . . . .	15
1.4.1	The Situation . . . . .	15
1.4.2	The First Round . . . . .	17
1.4.3	The Second Round . . . . .	21
1.4.4	After the Debate . . . . .	23
<b>2</b>	<b>Göttingen 1897–1910</b> . . . . .	27
2.1	Introduction . . . . .	27
2.2	Working in Applied Mathematics . . . . .	28
2.2.1	<i>Habilitation</i> . . . . .	29
2.2.2	Applied Mathematics After the <i>Habilitation</i> . . . . .	32
2.3	First Years as a <i>Privatdozent</i> in Göttingen . . . . .	33
2.3.1	<i>Privatdozenten</i> Grant and Titular Professorship . . . . .	33
2.3.2	Under Consideration in Breslau 1903 . . . . .	34
2.4	The Context: Hilbert’s Research on Foundations . . . . .	36
2.4.1	Mathematical Problems and Foundations . . . . .	36
2.4.2	Foundations of Geometry and Analysis . . . . .	40
2.4.3	The Paradoxes . . . . .	41
2.5	Starting Set Theory . . . . .	47
2.5.1	Zermelo’s First Period of Foundational Research . . . . .	47
2.5.2	Early Work . . . . .	48
2.5.3	The Heidelberg Congress 1904 . . . . .	50
2.6	Zermelo’s First Well-Ordering Proof . . . . .	53
2.6.1	The Proof and its Motivation . . . . .	53
2.6.2	Reactions . . . . .	59
2.7	Finite Sets . . . . .	61
2.8	The New Proof Paper . . . . .	65

2.8.1	Zermelo's Defence	65
2.8.2	The New Proof	69
2.8.3	Toward Acceptance of the Axiom of Choice	70
2.8.4	Postscriptum: Zermelo and Bernstein	72
2.9	The Axiomatization of Set Theory	76
2.9.1	Motivations	76
2.9.2	The Axiom System	79
2.9.3	Dedekind's Influence	84
2.9.4	The Axiom of Separation	86
2.9.5	The Range of the Axioms	89
2.9.6	Reception of the Axiom System	91
2.10	The Institutionalization of Mathematical Logic	92
2.10.1	Zermelo's Lectureship for Mathematical Logic	92
2.10.2	The 1908 Course "Mathematische Logik"	96
2.10.3	Hilbert and Zermelo	102
2.11	Waiting for a Professorship	105
2.11.1	Deterioration of Health	105
2.11.2	Würzburg 1906	106
2.11.3	Würzburg 1909	107
2.11.4	An Extraordinary Professorship	111
<b>3</b>	<b>Zurich 1910–1921</b>	<b>113</b>
3.1	A Full Professorship	113
3.1.1	Legends	113
3.1.2	The Chair in Zurich	114
3.1.3	Under Consideration at the Technical University of Breslau	117
3.1.4	Zermelo's Medical History	119
3.1.5	The End of Zermelo's Career	120
3.2	Colleagues and a Friend	125
3.3	Teaching in Zurich	128
3.4	Scientific Work in Zurich	129
3.4.1	Game Theory	129
3.4.2	Algebra and Set Theory	133
3.4.3	Ordinal Numbers	133
3.5	The Fraenkel Correspondence of 1921 and the Axiom of Replacement	135
<b>4</b>	<b>Freiburg 1921–1953</b>	<b>139</b>
4.1	A New Start	139
4.2	Research in Applied Mathematics	148
4.2.1	The Calculation of Tournament Results	148
4.2.2	The Zermelo Navigation Problem	150
4.2.3	On Splitting Lines of Ovals	152
4.3	The Return to the Foundations of Mathematics	153

4.3.1	The Return . . . . .	153
4.3.2	The Logic Project . . . . .	154
4.3.3	Cantor’s Collected Papers and the Relationship to Abraham Fraenkel . . . . .	158
4.3.4	The <i>Notgemeinschaft</i> Project . . . . .	163
4.4	Warsaw 1929 . . . . .	165
4.4.1	The Prehistory . . . . .	165
4.4.2	The Warsaw Programme . . . . .	167
4.4.3	The Nature of Mathematics . . . . .	169
4.4.4	Intuitionism <i>versus</i> Mathematics of the Infinite . . . . .	170
4.4.5	The Justification Problem . . . . .	171
4.5	Foundational Controversies: An Introduction . . . . .	175
4.6	Definiteness Revisited . . . . .	179
4.6.1	Definiteness Until 1930 . . . . .	179
4.6.2	Incorporating Definiteness Into the Notion of Set . . . . .	183
4.7	The Cumulative Hierarchy . . . . .	186
4.7.1	Genesis . . . . .	187
4.7.2	The Zermelo-Fraenkel Axiom System . . . . .	189
4.7.3	Cumulative Hierarchies, Large Cardinals, and the Paradoxes . . . . .	190
4.7.4	Continuation . . . . .	193
4.7.5	The Reception of the <i>Grenzzahlen</i> Paper . . . . .	194
4.8	The Skolem Controversy . . . . .	196
4.8.1	The Löwenheim-Skolem Theorem . . . . .	198
4.8.2	Skolem’s Paradox . . . . .	198
4.8.3	A “War” Against Skolem . . . . .	200
4.8.4	Modes of Defence . . . . .	202
4.9	Infinitary Languages and Infinitary Logic . . . . .	204
4.9.1	The Infinity Theses . . . . .	204
4.9.2	Infinitary Languages . . . . .	206
4.9.3	Infinitary Logic . . . . .	208
4.9.4	Infinitary Languages and the Cumulative Hierarchy . . . . .	209
4.9.5	The Failure . . . . .	210
4.10	The Gödel Controversy . . . . .	212
4.10.1	The Bad Elster Conference . . . . .	212
4.10.2	The Gödel Correspondence . . . . .	216
4.10.3	After the Controversy . . . . .	218
4.11	The Loss of the Honorary Professorship . . . . .	219
4.11.1	The University of Freiburg and Its Mathematical Institute After the Seizure of Power . . . . .	220
4.11.2	The Zermelo Case . . . . .	224
4.11.3	After the Renunciation . . . . .	231
4.12	Retreat . . . . .	236
4.12.1	Günterstal . . . . .	237
4.12.2	Last Scientific Activities . . . . .	243

4.12.3 The Time of Resignation .....	247
4.13 Twilight of Life .....	249
<b>5 A Final Word .....</b>	<b>259</b>
<b>6 Zermelo's <i>Curriculum Vitae</i> .....</b>	<b>263</b>
<b>7 Appendix</b>	
<b>Selected Original Versions .....</b>	<b>273</b>
7.1 Berlin .....	273
7.2 Göttingen .....	274
7.3 Zurich .....	282
7.4 Freiburg .....	287
<b>References .....</b>	<b>307</b>
<b>Index .....</b>	<b>339</b>