Problem 1

Let (M, g) be a Riemannian manifold, $c : [a, b] \to M$ a piecewise C^1 curve, and $\phi : [\tilde{a}, \tilde{b}] \to [a, b]$ a bijective and piecewise C^1 map. It follows that $\tilde{c} = c \circ \phi$ is also a piecewise C^1 curve. Show that $L^g(\tilde{c}) = L^g(c)$.

Problem 2

Let (M, g) be an n-dimensional Riemannian manifold, $f \in C^\infty(M)$, and $\phi : U^\phi \to V^\phi$ a chart. Show that:

$$\nabla f|_{U^\phi} = \sum_{j=1}^{n} \left(\sum_{i=1}^{n} g_\phi^{ij} \frac{\partial(f \circ \phi^{-1})}{\partial x^i} \circ \phi \right) \frac{\partial^\phi}{\partial x^j}$$

Here, $(g_\phi^{ij})_{i,j=1,\dots,n}$ is the inverse matrix of $(g^\phi_{ij})_{i,j=1,\dots,n}$, and $g^\phi_{i,j}(p) = g_p(\frac{\partial^\phi}{\partial x^i}\big|_p, \frac{\partial^\phi}{\partial x^j}\big|_p)$.