## Aufgabe

Let $b$ be a non-degenerate and symmetric bilinear form on $\mathbb{R}^{n+1}$ with $\mathrm{ind}_b = \nu$.

(a) Show that if $U$ is a subspace of $\mathbb{R}^{n+1}$ such that $b|_{U\times U}$ is negative definite, then $\dim_U = \nu$ if and only if $b|_{U^\perp \times U^\perp}$ is positive definite.

(b) Let $U$ be a subspace of $\mathbb{R}^{n+1}$ with $U \cap U^\perp = 0$. Then there exists an orthonormal basis $e_0, \dots, e_n$ of $\mathbb{R}^{n+1}$ such that $\mathrm{span}(e_0, \dots, e_k) = U$, $\mathrm{span}(e_{k+1}, \dots, e_n) = U^\perp$, and $b(e_i, e_j) = \epsilon_i \delta_{ij}$ for all $i, j \in 0, \dots, n$ where $\epsilon_i \in \{0, 1\}$.