

### Problem

Let  $\nu \in 1, \dots, n-1$  and let  $\langle v, w \rangle_\nu = -\sum_{i=1}^{\nu} v_i w_i + \sum_{i=\nu+1}^n v_i w_i$ ,  $v, w \in \mathbb{R}^n$ , be a symmetric, non-degenerate bilinear form on  $\mathbb{R}^n$  of index  $\nu$ . A tensor field  $b$  on  $T(\mathbb{R}^n) \simeq \mathbb{R}^n \times \mathbb{R}^n$  is defined by  $b_x(v, w) := \langle v, w \rangle_\nu$ . Show that:

There is no compact hypersurface  $M \subset \mathbb{R}^n$  such that the induced symmetric  $(2, 0)$ -tensor field  $i^*b$  of  $b$  on  $M$  is non-degenerate at every point  $x \in M$ .

*Hint: First consider that every hyperplane in  $\mathbb{R}^n$  appears as a tangent plane of  $M$ .*