

**Aufgabe** (4 Punkte)

Let  $(X, d)$  be a metric space.

1.  $B_r(x) = \{y \in X : d(x, y) < r\} \subset X$  is open and  $\overline{B_r(x)} = \{y \in X : d(x, y) \leq r\}$  is closed.
2. Let  $Y \subset (X, d)$ . The induced topology on  $Y$  coincides with the topology induced by the restricted metric  $d_Y := d|_{Y \times Y}$ .
3. Given a sequence  $(x_n)$  in a metric space,  $(x_n) \rightarrow x$  if and only if  $d(x_n, x) \rightarrow 0$  as  $n \in \mathbb{N} \rightarrow \infty$ .
4. A sequence cannot have more than one limit.
5. A point  $x \in X$  is an accumulation point of a set  $S \subset X$ , i.e.  $x \in \overline{S}$  ("x belongs to the closure of S"), if and only if there exists a sequence  $(x_n) \subset S$  such that  $x_n \rightarrow x$ . A set  $S \subset X$  is dense in  $X$  if  $\overline{S} = X$ .

Give an example for a metric space where  $\overline{B_r(x)} \neq \overline{B_r(x)}$ .

6. A map  $f : X \rightarrow Y$  between metric space is continuous if and only if  $(x_n) \rightarrow x$  in  $X$  implies  $f(x_n) \rightarrow f(x)$  in  $Y$ .