Task (4 points)

Let (X, d) be a metric space.

1. $B_r(x) = \{y \in X : d(x, y) < r\} \subset X$ is open, and $\overline{B}_r(x) = \{y \in X : d(x, y) \leq r\}$ is closed.

2. Let $Y \subset (X, d)$. The induced topology on Y coincides with the topology induced by the restricted metric $d_Y := d|_{Y \times Y}$.

3. Given a sequence (x_n) in a metric space, $(x_n) \to x$ if and only if $d(x_n, x) \to 0$ as $n \in \mathbb{N} \to \infty$.

4. A sequence cannot have more than one limit.

5. A point $x \in X$ is an accumulation point of a set $S \subset X$, i.e., $x \in \overline{S}$ ("x belongs to the closure of S"), if and only if there exists a sequence $(x_n) \subset S$ such that $x_n \to x$. A set $S \subset X$ is dense in X if $\overline{S} = X$.

   Provide an example of a metric space where $\overline{B}_r(x) \neq \overline{B_r(x)}$.

6. A map $f : X \to Y$ between metric spaces is continuous if and only if $(x_n) \to x$ in X implies $f(x_n) \to f(x)$ in Y.