

Problem

Let A and B be subsets of a metric space X and $r > 0$. Let d_H be the Hausdorff distance in X . Prove that

(a) $d_H(A, B) = \max\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\}$.

(b) $d_H(A, B) \leq r$ if and only if $d(a, B) \leq r \forall a \in A$ and $d(b, A) \leq r \forall b \in B$. This fails if one replaces \leq with $<$.