**Problem 1** (4 Punkte)
Let $(A, L)$ be a length structure on a topological Hausdorff space $X$ and let $d_L$ be the induced length metric. Show:

(a) $(X, d_L)$ is a metric space.

(b) Then any open set in $X$ is also open in $(X, d_L)$. Hence the topology induced by $d_L$ is finer than the original topology on $X$.

**Problem 2** (4 Punkte)
Let $(A, L)$ be a length structure on a topological Hausdorff space $X$.

(a) Any admissible path of finite length is continuous also w.r.t. to the length metric $d_L$.

(b) Show that $(X, d_L)$ is locally path connected, i.e. every neighborhood of any point contains a path connected neighborhood of this point (w.r.t. $d_L$).

(c) Consider the fan", i.e. $X = \bigcup_{n=1}^{\infty} X_i \cup [0,1] \times \{0\} \subset \mathbb{R}^2$ where $X_i = \{t(\cos(1/n), \sin(1/n)) : t \in [0,1]\}$. Show that the induced topology of $\mathbb{R}^2$ on $X$ cannot be the topology of length metric.

**Problem 3** (4 Punkte)
Prove that $\gamma : [0,1] \to \mathbb{R}^2$ given by

$$\gamma(t) = \begin{cases} (t, \pi t \sin(\frac{1}{t}) & t \in (0,1] \\ 0 & t = 0 \end{cases}$$

is continuous path but has infinite length w.r.t. the Euclidean length.

**Problem 4 (Bonus)** (2 Punkte)
Let $(A, L)$ be a length structure on a topological Hausdorff space $X$. We say two points $x, y \in X$ belong to the sam accesibility component if they can be connected by an admissible path of finite length. Show that

(a) Accessibility by paths is an equivalence relation.

(b) Accessibility components coincide with components of finiteness of $d_L$ (components of finiteness are the components that correspond to the equivalence relation in Problem 1 (a) on Sheet 2).

(c) Accessibility components are contained in path connected components (which agree with connected components) but in general are not equal to them.

*Abgabe am Mittwoch, 8. November bis 12 Uhr beim Assistenten.*