
Problem 1 (4 Punkte)
Let (X, d) be a metric space. Prove the following statements.

(a) If d is strictly intrinsic, then for all $x, y \in X$ there exists an α-midpoint $z \in X$ for all $\alpha \in (0, 1)$, i.e. $d(x, z) = \alpha d(x, y)$ and $d(z, y) = (1 - \alpha)d(x, y)$.

(b) Assume (X, d) is complete. If for all $x, y \in X$ there exists a ϵ-midpoint $z \in X$ for all $\epsilon > 0$, then d is intrinsic, i.e. a length metric.

Problem 2 (4 Punkte)
A complete metric space (X, d) is a length space if and only if given $\epsilon > 0$ and two points $x, y \in X$, there exists a finit sequence of points $x = x_1, \ldots, x_k = y$, $k \in \mathbb{N}$, such that $d(x_i, x_{i+1}) \le \epsilon$ for all $i \in \{1, \ldots, k-1\}$ and $\sum_{i=1}^{k-1} d(x_i, x_{i+1}) \le d(x, y) + \epsilon$. If (X, d) is strictly intrinsic, the last inequality becomes $\sum_{i=1}^{k-1} d(x_i, x_{i+1}) \le d(x, y)$.

Problem 3 (4 Punkte)

(a) Let X be a length space, and Y a metric space. Let $f : X \to Y$ be a locally Lipschitz map with Lipschitz constant $C > 0$. Prove that f is Lipschitz with the same Lipschitz constant.

(b) Let (X, d) be a length space and A a connected open subset of X. Then d induces on A a finite-valued intrinsic metric $d_A : A \times A \to [0, \infty)$. Moreover, every point $p \in A$ has a neighborhood $U \subset A$ such that for any two points $p, q \in U$ we have $d(p, q) = d_A(p, q)$.

Problem 4 (Bonus) (1 Punkte)
Prove that the completion of a length space is a length space.

Abgabe am Mittwoch, 22. November bis 12 Uhr beim Assistenten.