
Problem 1 (4 Punkte)
Let (X, d) be a metric space and let $\gamma : [a, b] \to X$ be a path. Show that the following two statements are equivalent.

1. γ is absolutely continuous (Lipschitz),
2. There exists $l \in L^1([a, b])$ ($l \in L^\infty([a, b])$) such that for all $s \le t \in [a, b]$ we have
$$d(\gamma(s), \gamma(t)) \le \int_s^t l(\tau) d\tau.$$

Problem 2 (4 Punkte)
Let (X, d) be a metric space and let $\gamma : [a, b] \to X$ be a path. Assume the speed $v_\gamma(t)$ exists for all $t \in [a, b]$ and is continuous in t. Prove that

$$L(\gamma) = \int_a^b v_\gamma(t) dt.$$

Problem 3 (4 Punkte)
Prove that for any path $\gamma : [a, b] \to X$, not necessarily simple, that

$$L(\gamma) = \sum_{k \in \mathbb{N} \cup \{\infty\}} k \mathcal{H}^1(\{x \in X : \#\gamma^{-1}(x) = k\})$$

where # denotes the cardinality of a set and we use the convention $0 \cdot \infty = 0$.
Hint: The right hand side is an integral of the function $x \mapsto \#\gamma^{-1}(x)$ *w.r.t.* $\mathcal{H}^1$. *Use the theorem of monotone convergence of integrals and recall* $\operatorname{diam} \gamma([a, b]) \le \mathcal{H}^1(\gamma([a, b])) \le L(\gamma)$.

Abgabe am Mittwoch, 22. November bis 12 Uhr beim Assistenten.