
Problem 1 (4 Punkte)
Prove that the metric d_C on $C(X)$ over a metric space (X, d) is intrinsic if and only if the metric d is intrinsic at distances less than π.

Problem 2 (4 Punkte)
Let (X, d) be a metric space with $\mathrm{diam}_X = \pi$. Suppose that $C(X)$ is a length space but X is not. Prove that there are three distinct points $x, y, z \in X$ such that $d(x, y) = d(y, z) = \pi$.

Problem 3 (4 Punkte)
Let (X, d) be the metric cone over segment of length L. Prove that

(a) (X, d) is nonpositively curved for any L.

(b) (X, d) is nonnegatively curved if and only if $L \leq \pi$.

Bonus Problem (4 Punkte)
Let $C(X)$ be the cone over a length space (X, d), and $\mathrm{diam}_X < \pi$. Prove that $C(X) = [0, \infty) \times_f X$ where $f(t) = t$.

Abgabe am Dezember, 06. November bis 12 Uhr beim Assistenten.