Problem 1 (4 Punkte)
Consider two length spaces X_1 and X_2 of nonpositive (nonnegative) curvature. Show that the direct metric product $X_1 \times X_2$ is a length space with nonpositive (nonnegative) curvature.
Hint: Consider a triangle Δ in $X_1 \times X_2$, and let Δ_1 and Δ_2 be the projections of Δ to X_1 and to X_2. Choose comparison triangles for Δ_1 and Δ_2 in $\mathbb{R}^2$ and construct a comparison triangle for Δ contained in a plane in $\mathbb{R}^2 \times \mathbb{R}^2$.

Problem 2 (4 Punkte)
Let $F \in C^2([0, L])$ and let $F'' = f$. A 1-Lipschitz function $g : [0, L] \to \mathbb{R}$ is called f-convex if $g - F$ is concave. Prove that

1. g is continuous.
2. g has right and left derivatives, and the left derivative is not greater than the right one.
3. The set of points where g is not differentiable is finite or countable.
4. The derivative of g is continuous on the set where it is defined.

Bonus Problem (6 Punkte)
Show that the following property is implied by but not equivalent to the triangle condition for nonpositive curvature. For any triangle Δabc, and any midpoints d and e of its sides $[ab]$ and $[bc]$ the inequality $2|de| \leq |ac|$ holds.
Hint: Consider a normed vector space. Abgabe am Dezember, 21. Dezember bis 12

Uhr beim Assistenten.