
Problem 1 (4 Punkte)
Prove that the First Variation Formula (4.21 Theorem) is valid for an arbitrary path $\gamma : [0, T] \to X$ (not necessarily a shortest path) as long as the direction of γ at $a = \gamma(0)$ is welldefined.

Problem 2 (4 Punkte)
Generalize the First Variation Formula (4.21 Theorem) to the case when both end-points of the shortest paths $\{\sigma_t\}_{t\in[0,T]}$ are variable and move along two shortest paths γ_1 and γ_2 with constant speeds v_1 and v_2. More precisely, prove that, if σ_t denotes a shortest path connecting $\gamma_1(t)$ and $\gamma_2(t)$ and a sequence $\{\sigma(t_i)\}_{i\in\mathbb{N}}$ converges to σ_0, then

$$\lim_{i\to\infty} \frac{l(t_i) - l(0)}{t_i} = -v_1 \cos\alpha_1 - v_2 \cos\alpha_2$$

where $l(t) = L(\sigma_t) = |\gamma_1(t)\gamma_2(t)|$ and $\alpha_1 = \cos\angle(\sigma_0\gamma_1)$ and $\alpha_2 = \cos\angle(\sigma_0^-\gamma_2)$ with $\sigma_0^- = \sigma_0(L(\sigma_0) - t)$.

Problem 3 (4 Punkte)
Formulate distance, angle and monotonicity definitions for spaces of curvature bounded from above (below) by k for any $k \in \mathbb{R}$. Prove the equivalence of these definitions.

Bonus Problem (4 Punkte)
Prove the following: If $k_1 > k_2$, then every space of curvature bounded from below by k_1 is a space of curvature bounded from below by k_2, and every space of curvature bounded from above by k_2 is a space of curvature bounded from above by k_1.

Abgabe am Donnerstag, 11. Januar bis 12 Uhr beim Assistenten.

Wir wüschen euch Frohe Weihnachten, eine erholsame Winterpause und einen guten Start ins neue Jahre!!!